Paper No. 984003
An ASAE Meeting Presentation

ANOTHER LOOK AT NONCONSTRAINED POST EMBEDMENT

by
Patrick M. McGuire, P.E.
Vice President
Borkholder Buildings
Nappanee, IN

Written for Presentation at the
1998 ASAE Annual International Meeting
Sponsored by ASAE

Disney's Coronado Springs Resort
Orlando, Florida
July 12-16, 1998

Summary:

The traditional formula for determining the required embedment depth for nonconstrained posts assumes that shear and moment at grade have the same algebraic sign, as is the case for determinant structures which are free to translate laterally (flag poles or billboard signs). This presentation demonstrates that with an indeterminant lateral force resistive system which consists of a combination of embedded posts and structural diaphragms (many buildings), the shear and moment at grade most often have opposite algebraic signs. An expression is derived for the critical eave deflection where base moments go from negative to positive.

Keywords:
embedment, nonconstrained, post, diaphragm, post frame, soil pressure, deflection

BACKGROUND

Traditionally, the derivation of embedment formulas begins with these assumptions:

1. The soil resistance to deformation is proportional to displacement.

2. The resistance to deformation increases linearly with depth below grade.

3. The post is rigid below grade. (Meador 1997)

Using these, the following equation for the depth below grade to the point of post rotation can be developed for a nonconstrained post. The derivation of this equation is beyond the scope of this brief paper, but it is readily available in the literature (Bohnhoff, 1992) (Meador, 1997).

$$\bar{y}=d\frac{(4M+3Vd)}{(6M+4Vd)} \quad (1)$$

Where:

$\bar{y}$ = depth below grade to the point of rotation;

V = shear at grade;

M = bending moment at grade;

d = depth of post embedment.

By an examination of (1), one can make the following observations. When:

$V = 0$ $\bar{y}$ = 2/3*d* Case 1;
M = 0 $\bar{y}$ = 3/4*d* Case 2;
M< 0 and V> 0 $\bar{y}$> 3/4*d* Case 3.

Prior to the emergence of post-frame diaphragm design methods, it was commonly assumed that the posts resisted all of the lateral wind loads. For such a structure, negative moments could not develop at grade so Case 3 could be neglected. This has given rise to the common misconception that $\bar{y}$ **always** varies from 2/3*d* to 3/4*d* for a nonconstrained post. $\bar{y}$ falls in this range only if *V* and *M* have the same sign. In a structure where lateral loads are resisted by both posts and a system of diaphragms, it is certainly possible that they do not.

Figure 1 shows the deflected shape of a post free to deflect at the eave (billboard) and Figure 2 shows the deflected shape of the same post restrained at the eave (post-frame building). Because of the indeterminant nature of the embedded post with eave restraint, it is obvious that the post **could** assume the deflected shape shown in Figure 2, but it is not obvious what shape it **will assume**.

PPSA4 MODEL

One straightforward way to determine the deflected shapes and soil pressure profiles each case generates is to analyze the two analogs shown in Figures 3 and 4 using a matrix analysis computer program such as the *Purdue Plane Structures Analyzer 4* (PPSA4). The soil is modeled as a series of "bars" or "springs." Their stiffness can be increased linearly with depth below grade by increasing their "area", just as the soil is assumed to.

It is necessary to have a "stiffness" of the soil, n_h. Both Bohnhoff (1992) and Meador (1997) developed soil stiffness values based on the work of earlier researchers. They assigned a range of values from 1000 pounds per cubic foot per foot below grade (pcf/ft) for soft clay to 40,000 pcf/ft for firm gravel (Meador 1997).

Meador (1997) states that a deflection *x* at a distance *y* below the ground surface results in a soil pressure of $n_h yx$. Thus, the total force on the first soil element from grade is:

$$P = n_h y_1 x b(d_{anlg}) \quad (2)$$

Where:

n_h = constant of horizontal soil reaction.

y_1 = depth below grade to the center of the first soil analog element.

d_{anlg} = arbitrary height of the soil element;

b = effective width of the post.

Applying the familiar equation for axial deflection of a bar to the first analog "bar" (of unit area) yields:

$$P = \frac{x(unit\ area)E_{anlg}}{L_{anlg}} \qquad (3)$$

Where:

E_{anlg} = an "equivalent" modulus of elasticity

L_{anlg} = arbitrary length of the soil analog "bar."

Equating (2) and (3) yields a modulus of elasticity of the soil analog element (using a set of consistent units):

$$E_{anlg} = \frac{n_h y_1 b(d_{anlg})(L_{anlg})}{(unit\ area)} \qquad (4)$$

Example

A nonconstrained building post measures 120 inches, *h*, from grade to eave. The post embedment, *d*, is 48 inches. The post is subjected to a uniform wind load of 10 pounds per linear inch of height. Two conditions were considered, one a post free to deflect at the eave (Analog 1), and two, a post completely laterally restrained at the eave by a stiff roof diaphragm (Analog 2). The post has an effective width, *b*, of 7.78" and an EI of 93,590,000 lbf(in)(in). A height of 8 inches was chosen for each soil analog element, with the depth to the center of the first element, y_1, of 4 inches. A length of 10 inches was chosen for the soil elements.

Analyses were performed for both Analogs 1 and 2 using n_h values of 1000, 5000, 10000 and 40000 pcf/ft. The corresponding E_{anlg} values are shown in Table 1. The calculated values for reactions in the soil analog elements and shear (Vg) and moment (Mg) at grade are presented in Graphs 1 and 2a through 2d. Soil pressure is directly proportional to these reactions. Graph 1 shows a reaction profile that is typical of the "traditional" nonconstrained post analyses. This result tends to confirm the validity of the analog.

For condition 2, current design practice is to assume that the soil pressure profile will be similar to the "traditional" profile, albeit for a smaller base moment. However, as the graphs show, this is not the case. For this example all of the base moments for condition 2 are negative. A "traditional" reaction profile was generated by applying the same shear with a positive moment at grade to Analog 3 (Figure 6). (Both reaction profiles are plotted as positive in Graphs 2a through 2d for ease of comparison.)

CRITICAL DEFLECTION AT THE EAVE

The deflected shape of the indeterminant post can also be determined by recognizing that when moment at grade equals zero, the shear at both the eave restraint and grade will equal half of the applied uniform load. One can solve for the eave deflection required for this condition to develop by applying the principle of superposition as shown in Figure 5. First the post is analyzed under uniform wind loading. Second the post is analyzed for a concentrated restraining force at the eave. Finally the critical deflection can be found by summing the component deflections.

First, consider the deflection at grade of the unrestrained post using a relationship developed by Meador (1997, Equation 64):

$$\Delta = \frac{6(4M + 3Vd)}{n_h b d^3} \qquad (5)$$

Where:

Δ = the lateral deflection at grade of a post without eave restraint (Figure 1).

For a post with uniform wind pressure:

$$M_w = \frac{wh^2}{2} \tag{6}$$

$$V_w = wh \tag{7}$$

Where

w = the uniform wind load (pounds per inch) against the post

Substituting (6) and (7) into (1) yields:

$$\bar{y}_w = d\left(\frac{2h+3d}{3h+4d}\right) \tag{8}$$

Determine deflection at grade using (5):

$$\Delta_w = \frac{6wh}{n_h b d^3}(2h+3d) \tag{9}$$

The deflection at the eave, due to rotation below grade is then:

$$\Delta e_1 = \Delta_w \frac{(h+\bar{y}_w)}{\bar{y}_w} \tag{10}$$

Substituting (8) and (9) into (10) yields:

$$\Delta e_1 = \frac{6wh}{n_h b d^4}(3h^2+6dh+3d^2) \tag{11}$$

There is also an elastic component to the deflection at the eave (Figure 1). From any standard engineering text the formula for this deflection (neglecting shear) is:

$$\Delta e_2 = \frac{wh^4}{8EI} \tag{12}$$

Where:

E = the modulus of elasticity of the post

I = the moment of inertia of the post

The deflections, δe, due to the restraining force, *Re*, can be derived similarly:

$$V_r = Re = \frac{wh}{2} \tag{13}$$

$$M_r = Reh = \frac{wh^2}{2} \tag{14}$$

Substituting (13) and (14) into (1) yields:

$$\bar{y}_r = d\left(\frac{2h+1.5d}{3h+2d}\right) \tag{15}$$

Determine deflection at grade using (5):

$$\Delta_r = \frac{6wh}{n_h b d^3}(2h+1.5d) \tag{16}$$

The deflection at the eave, due to rotation below grade is then:

$$\delta e_1 = \Delta_r \frac{(h+\bar{y}_r)}{\bar{y}_r} \tag{17}$$

Substituting (15) and (16) into (17) yields:

$$\delta e_1 = \frac{6wh}{n_h b d^4}(3h^2+4dh+1.5d^2) \tag{18}$$

The elastic deflection due to restraining force at the eave is:

$$\delta e_2 = \frac{Reh^3}{3EI} = \frac{wh^4}{6EI} \tag{19}$$

The critical deflection is then:

$$\Delta_{cr} = \Delta e_1 + \Delta e_2 - \delta e_1 - \delta e_2 \tag{20}$$

Substituting (11), (12), (18) and (19) into (20) yields:

$$\Delta_{cr} = \frac{3wh}{n_h b d^3}(4h+3d) - \frac{wh^4}{24EI} \qquad (21)$$

For eave deflections less than the critical deflection, negative moments will develop at the base. For eave deflections greater than the critical deflection, positive moments will develop at the base. A word of caution is necessary. As the required soil stiffness becomes higher, the moment at grade becomes more sensitive to original assumption number three, that the post is rigid below grade. This assumption is not made when a matrix analysis program is used. Such analysis gives a slightly greater critical eave deflection than when deflections below grade are neglected.

For the previous example, Table 1 also presents the critical eave deflections calculated using (21) and PPSA4.

CONCLUSIONS

When a nonconstrained post is supported above grade by a diaphragm, the structure becomes indeterminant. Often the shear and moment at grade will not act in the sense that is assumed by the traditional nonconstrained embedment formula. Current design practices should be revised in recognition of this.

FURTHER STUDY

Current design practice is to increase the allowable lateral soil bearing pressure by a factor of two, when the post is isolated. It is unclear if rather than increasing the allowable soil pressure, one should increase the effective width of the post, *b*. Increasing *b* increases the lateral stiffness of the embedded post, where as current practice does not.

ACKNOWLEDGEMENT

This author gratefully acknowledges the advice and encouragement of Dr. Gary Anderson, South Dakota State University.

REFERENCES

Bohnhoff, D.R., 1992, Estimating frame stiffness and eave loads for diaphragm analysis of post-frame buildings, *Trans. ASAE*, 35(3):1043-1054.

Meador, N.F., 1997, Mathematical models for lateral resistance of post foundations, *Trans. ASAE*, 40(1):191-201.

McGuire, P.M., 1998, Overlooked assumption in nonconstrained post embedment, *Prac. Per. On Struct. Des. Const.*, ASCE, 3(1):19-24.

TABLE 1

n_h, soil stiffness	E_{anlg}, soil element 'E' *	Δ_{cr}, per (21) +	Δ_{cr}, PPSA4 +
1000 pcf/ft	120 psi	53.216"	53.826"
5000 pcf/ft	600 psi	9.905"	10.184"
10,000 pcf/ft	1200 psi	4.490"	4.740"
40,000 pcf/ft	4800 psi	0.430"	0.650"

* b = 7.78", d_{anlg} = 8", y_1 = 4", L_{anlg} = 10"

+ h = 120", d = 48", w = 10 pli,
EI = 93.59E6 lbf(in^2)

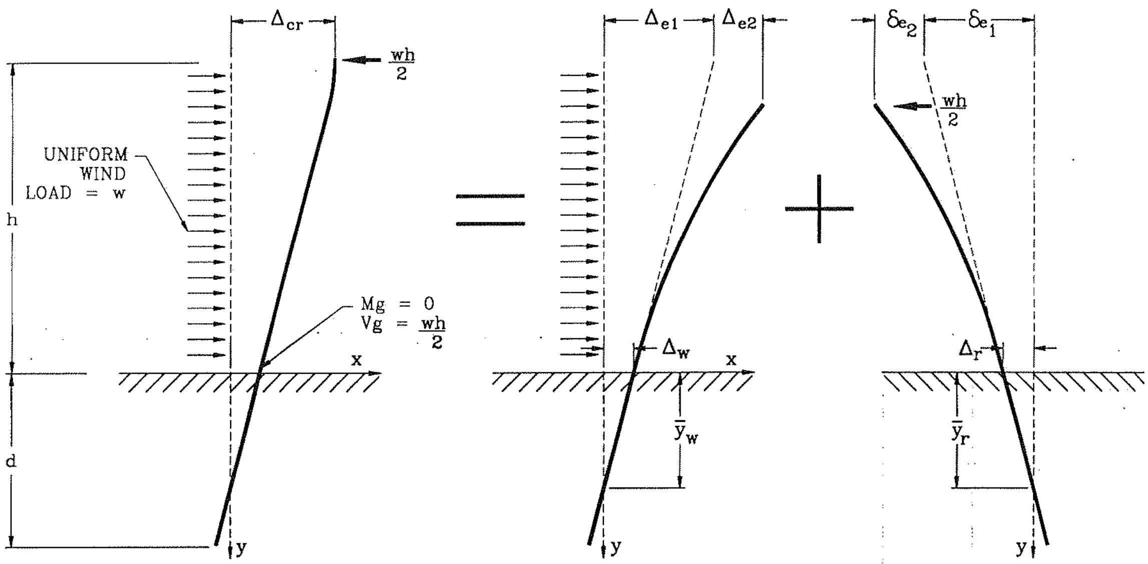


Figure 5. Nonconstrained Post at Critical Deflection
Moment at Grade, Mg = 0

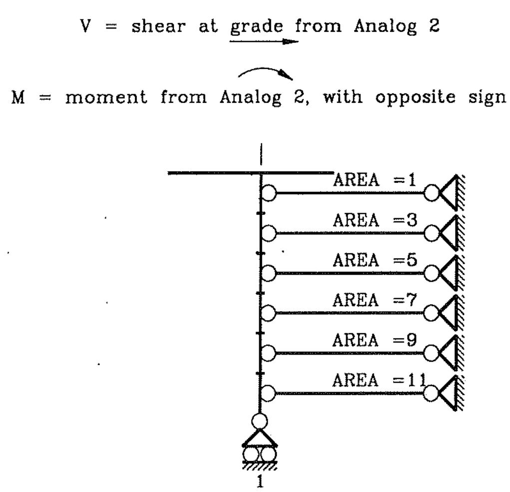
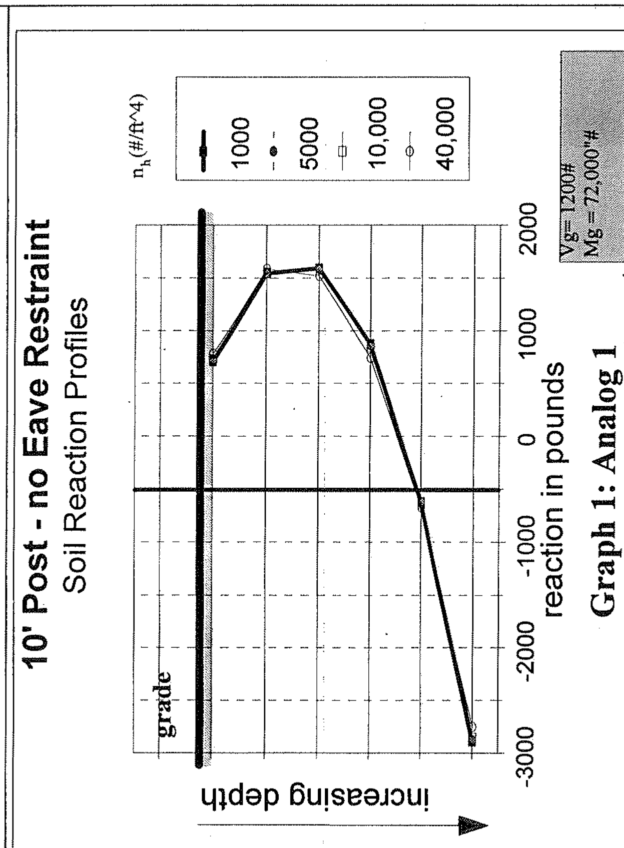
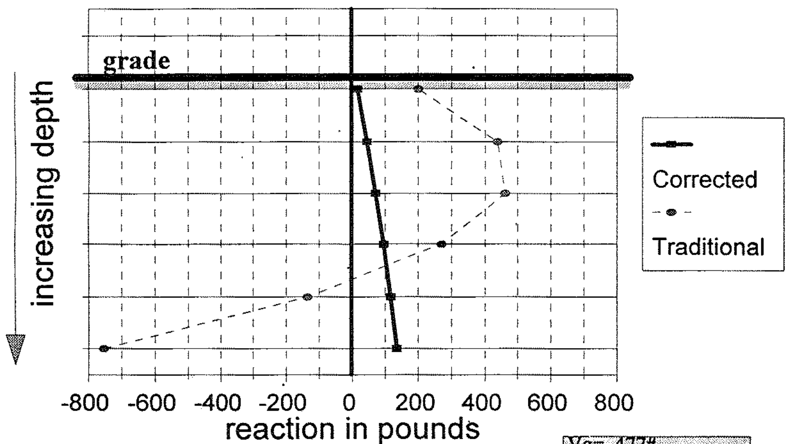


Figure 6. – Analog used to Generate
'Traditional' Soil Reaction Profiles



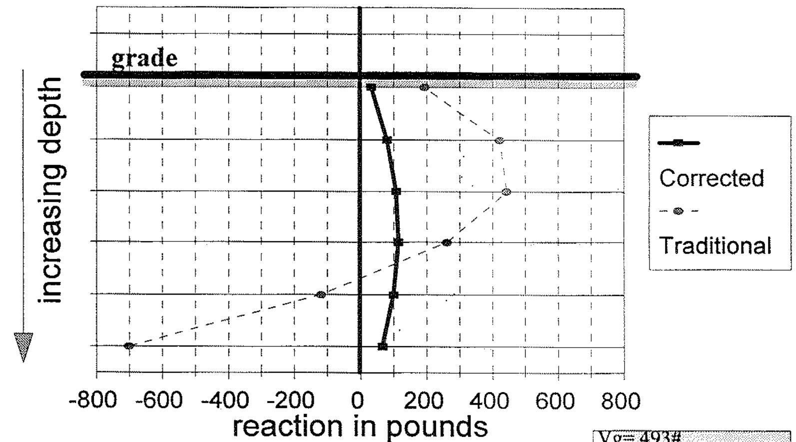
10' Post Restrained at Eave
Soil Reaction Profiles nh = 1000



Vg= 477#
Mg = -14,816"#

Graph 2a: Analog 2

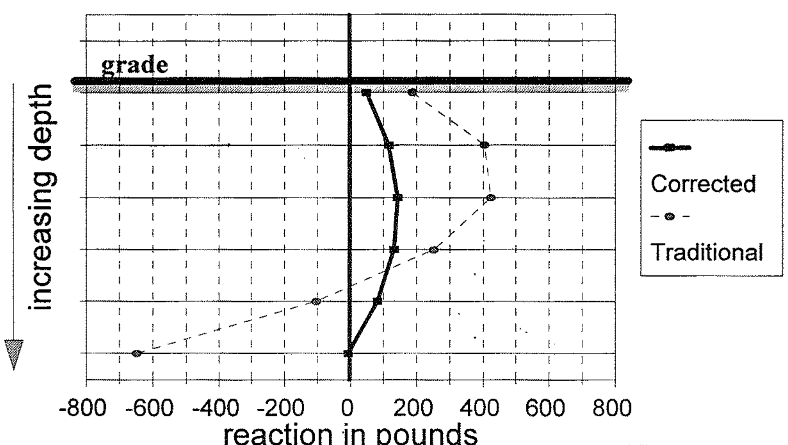
10' Post Restrained at Eave
Soil Reaction Profiles nh = 5000



Vg= 493#
Mg = -12,788"#

Graph 2b: Analog 2

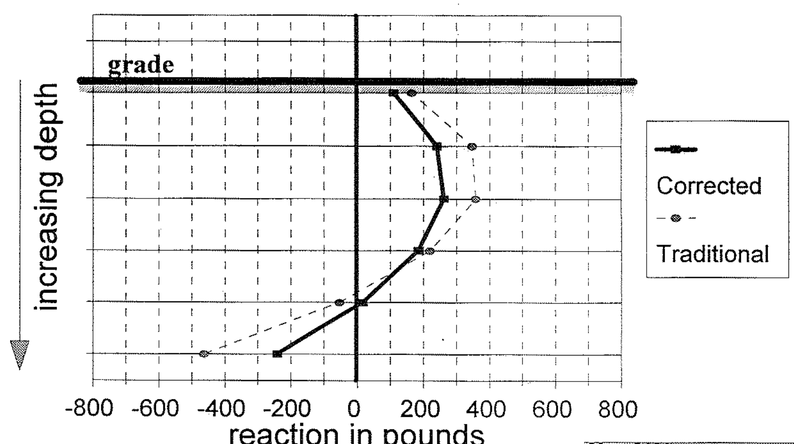
10' Post Restrained at Eave
Soil Reaction Profiles nh = 10,000



Vg= 511#
Mg = -10,710"#

Graph 2c: Analog 2

10' Post Restrained at Eave
Soil Reaction Profiles nh = 40,000



Vg= 569#
Mg = -3,681"#

Graph 2d: Analog 2