Overlooked Assumption in Nonconstrained Post Embedment

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Abstract: In the traditional formula for determining the required embedment depth for nonconstrained posts it is assumed that shear and moment at grade have the same algebraic sign, as is the case for determinant structures that are free to translate laterally (flag poles or billboard signs). This article demonstrates that in an indeterminant lateral force resistive system, which consists of a combination of embedded posts and structural diaphragms (many buildings), the shear and moment at grade most often have opposite algebraic signs. In this case the traditional formula should not be applied. This article presents an alternate method for checking embedment depth based upon the calculated lateral soil bearing pressure.

INTRODUCTION

The formula that has traditionally been used for determining the depth of embedment of wood posts required to resist lateral loads where no constraint is provided at the ground surface, such as a rigid floor or rigid ground surface pavement, is

$$d = 0.5A[1 + (1 + (4.36h/A))^{1/2}] \qquad (1)$$

where

$$A = 2.34P/S_1b \qquad (2)$$

In these equations, b = diameter of round post or footing or diagonal dimension of square post; d = depth of embedment in earth in feet (meters), not over 12 ft (3,658 mm) for computing lateral pressure; h = distance in feet (meters) from ground surface to point of application of P; P = applied lateral force in pounds (kilonewtons); and S_1 = allowable lateral soil-bearing pressure (pounds per square foot per foot of depth) based on a depth of one-third the depth of embedment (kilopascals).

This formula has served at least two generations of engineers very well and was included in the May 1997 working draft of the new International Building Code (IBC) under Section 1805.7.2.1. However, it is the opinion of this author that a further limitation should be applied to the use of this equation: "that no lateral constraint is provided above the ground surface, such as a structural diaphragm." It is the purpose of this paper to explain the author's rationale for adding this limitation and to offer an alternate method for checking the embedment of nonconstrained posts.

BACKGROUND

Since the 1970s there has been an increasing interest in the structural design of low-rise buildings using lateral force resistive systems (LFRS), which consist of a combination of frames and diaphragms. A leading researcher of steel frames combined with diaphragms is Larry Luttrell, who wrote a design manual published by the Steel Deck Institute (1987). A vast amount of research has been conducted on this subject in the wood building industry, primarily through the American Society of Agricultural Engineers.

Since the time the nonconstrained embedment formula first appeared, about 35 years ago, buildings using preservatively treated wood posts as structural members have evolved significantly (Bender 1992). Today most "post-frame building" LFRS are designed using a combination of embedded posts and roof/wall diaphragms (Gebremedhin and Manbeck 1992; Woeste et al. 1992). To design these combined LFRS correctly, one must recognize the assumptions inherent in the derivation of the nonconstrained formula. This paper shows that one (commonly overlooked) assumption is that the top of the post is free to translate. When a system of roof and wall diaphragms provide lateral restraint above grade, this overlooked assumption is violated and the nonconstrained formula gives erroneous results.

REVIEW OF DESIGN PROCEDURES

Several authors, most recently Meador (1997), have derived formulas similar to the one in the IBC. They begin with these assumptions:

1. The soil resistance to deformation is proportional to displacement.
2. The resistance to deformation increases linearly with depth below grade.
3. The post is rigid below grade (Meador 1997).

They use these assumptions to develop the following equation for the depth below grade to the point of post rotation as a function of shear and moment at grade. The derivation of this equation is beyond the scope of this brief paper, but it is readily available in the literature (Bohnhoff 1992; Meador 1997).

$$\bar{y} = d\,\frac{(4M + 3Vd)}{(6M + 4Vd)} \qquad (3)$$

where $\bar{y}$ = depth below grade to the point of rotation; V = shear at grade; and M = bending moment at grade.

It is at this stage that the overlooked fourth assumption is made: that shear and moment at grade have the same algebraic sign (i.e., they act in the same sense). Once this assumption is made, formulas similar to (1) are derived. However, by examining (3), one can make the following observations:

$V = 0$	$\bar{y} = 2/3d$	case 1
$M = 0$	$\bar{y} = 3/4d$	case 2
$M < 0$ and $V > 0$	$\bar{y} > 3/4d$	case 3

Perhaps because the early researchers were primarily concerned with determinant structures, they neglected case 3. This has given rise to the common misconception that $\bar{y}$ *always* varies from $2/3d$ to $3/4d$ for a nonconstrained post. However, $\bar{y}$ falls in this range only if V and M have the same sign. It is certainly possible that they do not.

The key to understanding this is in the deflected shapes of the posts, shown in Figs. 1 and 2. Fig. 1 shows the deflected

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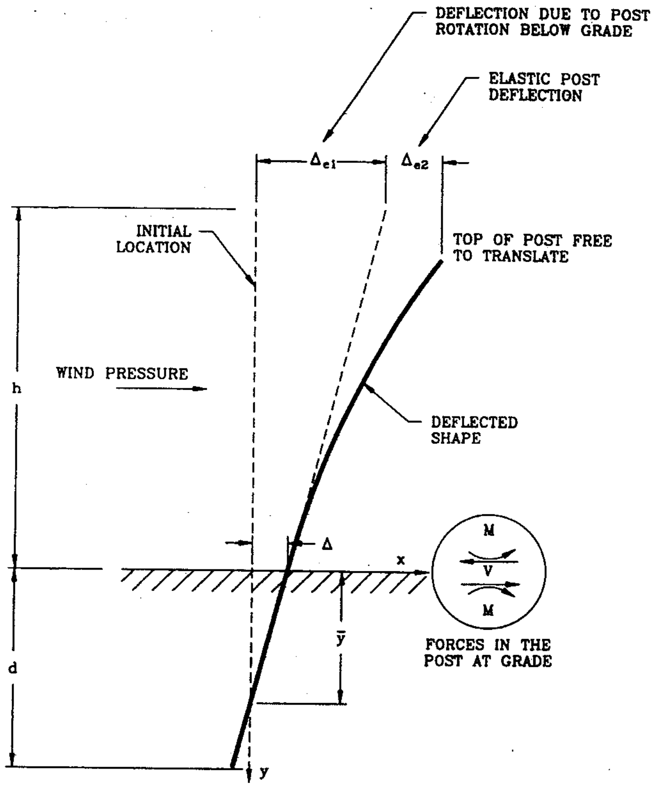


FIG. 1. Post Nonconstrained at Grade, Unrestrained at Eave

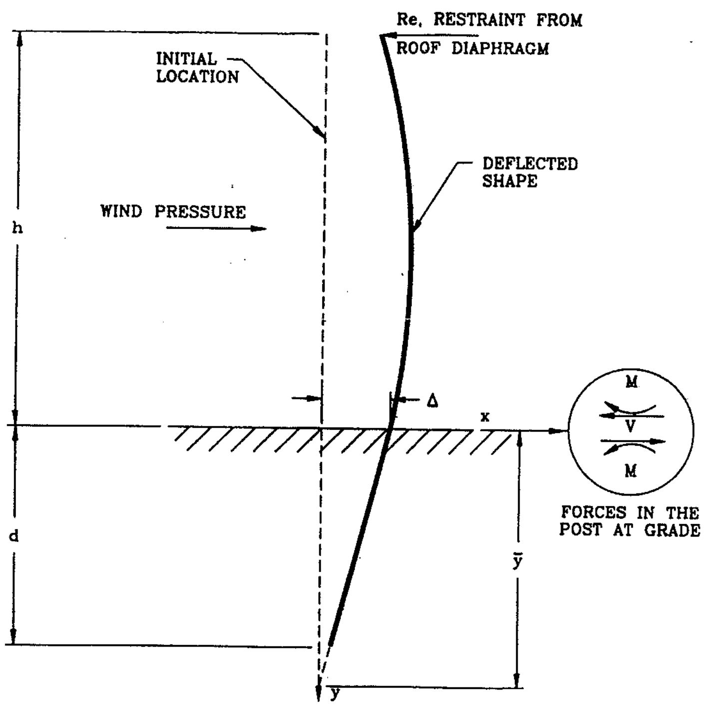


FIG. 2. Post Nonconstrained at Grade, Restrained at Eave

shape assumed by (1) (billboard) and Fig. 2 shows the deflected shape of the same post restrained at the eave (post-frame building). Part of the appeal of (1) is that it is determinate. The soil pressure depends only on the embedment and the applied loads, not the soil properties or the stiffness of the post. In contrast, a combined LFRS is indeterminant. That is, the soil pressure and forces at grade vary with the relative stiffnesses of soil, post, and diaphragms. A weakness in the traditional methodology is that it provides no guidance for calculating deflections. In contrast, the methods developed by Bohnhoff and Meador and presented in this paper allow a designer to estimate deflections. Because of the indeterminant nature of the embedded post with eave restraint, it is obvious that the post *could* assume the deflected shape shown in Fig. 2, but it is not obvious what shape it *will* assume.

Empirical Analog

One straightforward way to determine the deflected shapes and soil pressure profiles each case generates is to analyze the two analogs shown in Figs. 3 and 4 using a matrix analysis computer program such as the *Purdue Plane Structures Analyzer 4* (PPSA4). The soil is modeled as a series of "bars" or "springs." The stiffness of these bars or springs can be increased linearly with depth below grade by increasing their "area," just as the soil is assumed to by (1). It is necessary to have a stiffness value of the soil, n_h. Both Bohnhoff (1992) and Meador (1997) developed soil stiffness values based on the work of earlier researchers. They assigned a range of values from 1,000 pounds per cubic foot per foot below grade (pcf/ft) for soft clay to 40,000 pcf/ft for firm gravel (Table 1). A modulus of elasticity of the soil analog element can be calculated (using a set of consistent units).

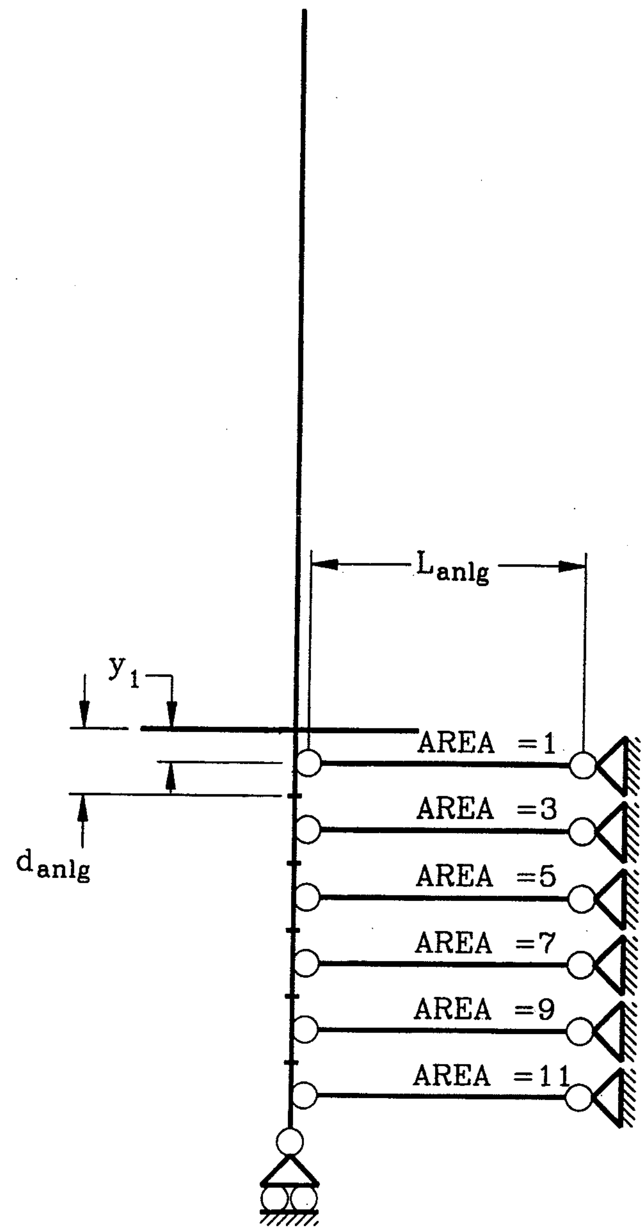
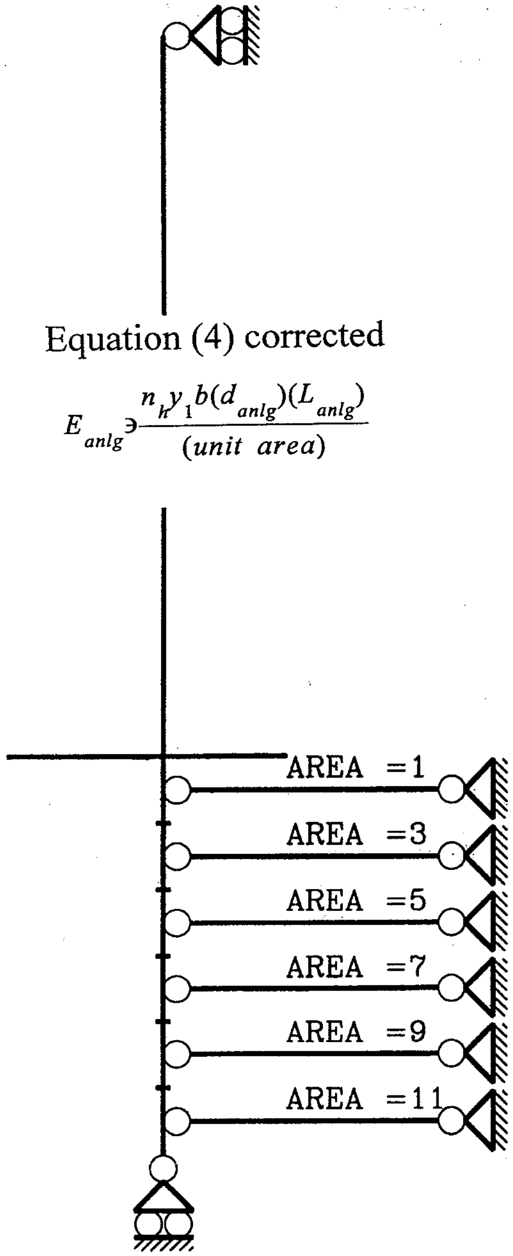


FIG. 3. Analog 1 Post Nonconstrained and Unrestrained



$$E_{anlg} \ni \frac{n_h y_1 b(d_{anlg})(L_{anlg})}{(unit\ area)}$$

FIG. 4. Analog 2 Post Nonconstrained and Restrained

$$E_{anlg} = \frac{n_h y_1 b(d_{anlg})}{L_{anlg}} \tag{4}$$

where E_{anlg} = "equivalent" modulus of elasticity; n_h = constant of horizontal soil reaction; y_1 = depth below grade to center of first soil analog element; d_{anlg} = arbitrary depth (or height) of soil analog; and L_{anlg} = arbitrary length of soil analog "bar."

Limiting Value of Constant of Horizontal Soil Reaction

The deflected shape of the indeterminant post can also be determined by recognizing that when moment at grade equals zero, the shear at both the eave restraint and grade equal half of the applied uniform load. One can solve for the minimum soil stiffness required for this condition to develop by applying the principle of superposition to an unrestrained post, as shown in Fig. 1. First, the post is analyzed under uniform wind loading. Second, the post is analyzed for a concentrated restraining force at the eave. Finally, the critical value of soil stiffness is found by applying the compatibility condition that deflections at the eave sum to zero. First, consider the deflection at grade of the unrestrained post using another relationship developed by Meador (1997, Eq. 64):

TABLE 1. Presumed Soil Properties for Post Foundation Design (in Absence of Codes or Tests) and Constants of Horizontal Soil Reaction

Class (1)	Description of Materials (2)	Density (3)	Estimated Constant of Horizontal Soil Reaction (kPa/m⁴) (4)	(lbf/ft⁴) (5)
3	Sandy gravel or gravel	Firm	6,285	40,000
3	Sandy gravel or gravel	Loose	1,570	10,000
4	Sand, silty sand, clayey sand, silty gravel, and clayey gravel	Firm	1,570	10,000
4	Sand, silty sand, clayey sand, silty gravel, and clayey gravel	Loose	1,180	7,500
5	Clay, sandy clay, silty clay and clayey silt	Medium	785	5,000
5	Clay, sandy clay, silty clay and clayey silt	Soft	160	1,000

Note: Values from Meador (1997, Table 2).

$$\Delta = \frac{6(4M + 3Vd)}{n_h b d^3} \tag{5}$$

where Δ = lateral deflection at grade of post without eave restraint (Fig. 1).

For a post with uniform wind pressure,

$$M_w = \frac{wh^2}{2} \tag{6}$$

$$V_w = wh \tag{7}$$

where w = uniform wind load (pounds per inch) against post.

Substituting (6) and (7) into (3) yields

$$\bar{y}_w = d\left(\frac{2h + 3d}{3h + 4d}\right) \tag{8}$$

Determine deflection at grade using (5).

$$\Delta_w = \frac{6wh}{n_h b d^3}(2h + 3d) \tag{9}$$

The deflection at the eave, due to rotation below grade, is then

$$\Delta e_1 = \Delta_w \frac{(h + \bar{y}_w)}{\bar{y}_w} \tag{10}$$

There is also an elastic component to the deflection at the eave (Fig. 1). In any standard engineering text the formula for this deflection (neglecting shear) is

$$\Delta e_2 = \frac{wh^4}{8EI} \tag{11}$$

where E = modulus of elasticity of post; and I = moment of inertia of post.

Second, the deflections, δe, due to the restraining force, Re, applied to the eave of the post shown in Fig. 1 can be derived similarly, where

$$V_R = Re = \frac{wh}{2} \tag{12}$$

$$M_R = Reh = \frac{wh^2}{2} \tag{13}$$

Note that $\bar{y}_R$ and Δ_R must be calculated on the basis of M_R and V_R. The elastic deflection is

$$\delta e_2 = \frac{Reh^3}{3EI} = \frac{wh^4}{6EI} \tag{14}$$

where δe_2 = elastic deflection due to restraining force at the eave. With a rigid restraint at the eave, the summation of deflections must equal zero.

$$\Delta e_1 + \Delta e_2 - \delta e_1 - \delta e_2 = 0 \qquad (15)$$

The following equation can be derived by substituting terms and algebraic manipulation:

$$n_{h0} = \frac{72EI}{bd^3h^3}(3d + 4h) \qquad (16)$$

where n_{h0} = minimum soil stiffness for moment at grade M to equal zero.

For values of $n_h < n_{h0}$ the bending moment at grade is less than zero ($M < 0$) and the deflected shape of the post is as shown in Fig. 2. A word of caution is necessary. As the required soil stiffness becomes higher, the moment at grade becomes more sensitive to the original assumption that the post is rigid below grade. This assumption is not made when a matrix analysis program is used to analyze analog 2 (Fig. 4). Such analysis gives a greater required soil stiffness than when deflections below grade are neglected.

For $n_h > n_{h0}$ the bending moment is greater than zero ($M > 0$). In this circumstance, a point of inflection will be present in the post, and the deflected shape below the point of inflection will be similar to Fig. 1. In this author's opinion, this will be very rare in practice. Although the IBC nonconstrained formula could apply to this case, the method that will be presented later is general enough to apply to this case as well.

Calculating Horizontal Soil Pressure

At this point there is a way for the designer to determine the deflected shape of the post. The preceding equations have shown the necessity of an alternate method to check embedment when the shape is as shown in Fig. 2. The equation for soil pressure was developed by Meador (1977): In brief, for a rigid post the deflection below grade, x, is along the straight line defined by

$$x = \Delta - \frac{\Delta}{\bar{y}}y = \Delta\left(1 - \frac{y}{\bar{y}}\right) \qquad (17)$$

where y = distance below grade.

As already noted, the soil stiffness measured by n_h is commonly taken to increase linearly with depth—that is, n_hy. The soil pressure, q, is then the soil stiffness times the distance below grade times the deflection.

$$q = n_hy(x) \qquad (18)$$

Substituting for x yields

$$q = n_h\Delta\left(y - \frac{y^2}{\bar{y}}\right) \qquad (19)$$

By setting the summation of forces at grade equal to zero

$$V = b\int q = n_h\Delta b\left(\frac{y^2}{2} - \frac{y^3}{3\bar{y}}\right)\Bigg]_0^d \qquad (20)$$

Since $q = 0$ at $y = 0$, the constant of integration equals zero. By evaluating the integral and algebraic manipulation, the following equation for Δ can be derived:

$$\Delta = \frac{V}{\left(\frac{d^2}{2} - \frac{d^3}{3\bar{y}}\right)n_hb} \qquad (21)$$

Once Δ is determined using (21) or (5), it can be substituted into (19) and the soil pressure at any point below grade can be determined.

Calculating Required Soil Strength

As shown in the examples, there are now two curves under consideration (Figs. 5 and 6). Eq. (19) plots the pressure imposed on the soil by the post, q. The second curve is the straight line S_ry, where S_r = required soil strength. Meador quotes Terzaghi, who stated that the required soil strength can be determined when the slope of the soil pressure curve equals the slope of the required strength curve at $y = 0$ (grade). This statement can be more easily understood through an examination of Figs. 5 and 6 that accompany the example. The required soil strength is *not* the maximum pressure divided by the depth to that pressure. Instead, the required soil strength increases as depth decreases. As Terzaghi noted, the pressure curve for this case is steepest at grade. Meador notes that by setting the two slopes equal at grade

$$S_r = n_h\Delta \qquad (22)$$

Meador also points out that (for the determinant case) it has been traditional to use an average resisting pressure by defining S_r at some distance below grade (such as S_1 in the IBC draft). This results in calculated pressures, q, that exceed S_ry for some regions of the post below grade. A discussion of this subject is beyond the scope of this paper. Therefore, the values of S_r presented in the examples are those where q will not exceed S_ry.

At this stage, the paper has presented all the tools necessary to check the embedment of a nonconstrained post in a combined LFRS. Remember that since this is an indeterminant system, the shear and moment at grade must be determined based on the relative stiffnesses of the diaphragms, posts, and soil. If the stiffness of the soil and the depth of embedment are considered during the lateral wind load analysis of the post-frame building, then the values of M, V, and $\bar{y}$ will vary

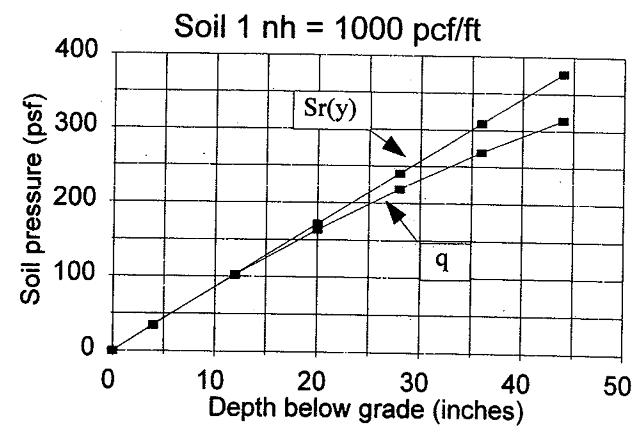


FIG. 5. Graph 1

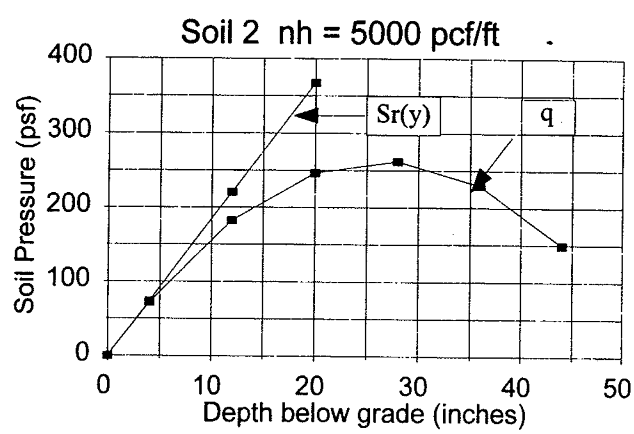


FIG. 6. Graph 2

with n_h. Therefore this author considers it most appropriate to select a trial embedment depth, determine M and V, and check S_r. One excellent method to determine shear and moment at grade was presented by Bohnhoff (1992). In the two examples that follow, it is assumed that lateral deflection at the top of the post is negligible. This situation could arise where posts are located close to a shear wall (Gebremedhin 1992).

EXAMPLES

A nonconstrained building post measures 120 in. (h) from grade to eave. The post embedment, d, is 48 in. The post is subjected to a uniform wind load of 10 lbs. per linear inch of height. The top of the post is completely laterally restrained at the eave by a stiff roof diaphragm. The post has an effective width, b, of 7.78 in. and an EI of 89,520,000 lbf(in.)(in.). First estimate n_{h0}, using (16).

$$n_{h0} = \frac{(72)(89{,}520{,}000)}{(7.78)(48^3)(120^3)}(3(48) + 4(120))(12^4)$$

$$n_{h0} = 56{,}094(\text{pcf/ft})$$

A trial-and-error analysis using analog 2 (Fig. 4) and PPSA4, which includes post deflections below grade, gives an estimate of the critical soil stiffness of 73,100 pcf/ft. Both of these values are well beyond the upper range of soil stiffnesses presented in Table 1. So the post will assume the shape shown in Fig. 2.

Soft Clay Soil

Soil 1 is a very soft clay with a constant of horizontal soil reaction, n_h, of 1,000 lbs per cubic foot per foot (0.048 lbs per cubic in. per in.). By engineering analysis it has been determined that shear at grade, V, is 475.3 lbs and the bending moment at grade, M, is −14966.1 in.-lbs.

The depth to point of rotation can be determined using (3).

$$\bar{y} = (48)\frac{4(-14966.1) + 3(475.1)(48)}{6(-14966.1) + 4(475.1)(48)} = 288.48 \text{ in.}$$

The deflection at grade can be determined using (21).

$$\Delta = \frac{475.1}{\left(\frac{48^2}{2} - \frac{48^3}{3(288.48)}\right)(0.048)(7.78)} = 1.24 \text{ in.}$$

The required soil strength can be determined using (22).

$$S_r = (0.048)(1.24)(12^3) = 102.9 \text{ psf/ft}$$

The calculated lateral deflection at grade using PPSA4 and analog 2 was 1.28 in. The soil pressures at various depths can be calculated using (19). The calculated pressures also agreed well with the reactions in the "springs" of analog 2. The pressure profile and $S_r y$ is presented in Fig. 5.

Medium Clay Soil

Soil 2 is medium clay with a constant of horizontal soil reaction, n_h, of 5,000 lbs per cubic foot per foot (0.241 lbs per cubic in. per in.). By engineering analysis it has been determined that shear at grade, V, is 490.9 lbs and the bending moment at grade, M, is −13089.8 in.-lbs.

Using (3) yields

$$\bar{y} = 56 \text{ in.}$$

Using (21) yields

$$\Delta = 0.53 \text{ in.}$$

Using (22) yields

$$S_r = (0.241)(0.530)(12^3) = 220.7 \text{ psf/ft}$$

The PPSA4 analysis gave the calculated deflection at grade as 0.55 in. Calculated pressures agreed well and are plotted in Fig. 6.

CONCLUSIONS

When a nonconstrained post is supported above grade by a diaphragm, the structure becomes indeterminant. Often the shear and moment at grade will not act as assumed in the traditional nonconstrained embedment formula. In those cases it is necessary to check lateral embedment by calculating the pressure imposed by the post on the soil. A designer must resist the temptation to consider the traditional formula conservative. Remember that load goes to the *stiffer* structural element, not to the stronger. It is dangerous to simplify design by assuming a determinant structure rather than performing an indeterminant analysis. No amount of embedment depth can compensate for an improperly designed diaphragm.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

- b = diameter of round post or footing or diagonal dimension of square post;
- d = depth of embedment in earth in feet (m);
- d_{anlg} = arbitrary depth (or height) of soil analog;
- E = modulus of elasticity of post;
- E_{anlg} = "equivalent" modulus of elasticity of soil analog element;
- h = distance in feet (m) from ground surface to point of application of P;
- I = moment of inertia of post;
- L_{anlg} = arbitrary length of soil analog element;
- M = bending moment at grade;
- n_h = constant of horizontal soil reaction;
- n_{h0} = minimum soil stiffness for moment at grade, M, to equal zero;
- P = applied lateral force in pounds (kilonewtons);
- q = soil pressure (psf);
- Re = restraining force applied to post at eave;
- S_r = the required soil strength;
- S_1 = allowable lateral soil-bearing pressure (pounds per square foot per foot of depth) based on depth of one-third depth of embedment (kilopascals);
- V = shear at grade;

w = the uniform wind load (lbs/in.) against the post;
x = lateral post deflection below grade;
y = distance below grade;
$\bar{y}$ = depth below grade to point of rotation;
y_1 = depth below grade to center of first soil analog element;
Δ = lateral deflection at grade of post without eave restraint;
Δe = deflection at eave of post without lateral restraint at eave due to uniform wind load; and
δe = deflection at eave of post without lateral restraint at eave due to restraining force applied at eave.

Subscripts

R = produced by concentrated restraining force applied to post without lateral restraint at eave;
w = produced by uniform lateral wind load along length (above grade) of post without lateral restraint at eave;
1 = deflection of post without lateral restraint at eave caused by post rotation below grade; and
2 = deflection of post without lateral restraint at eave caused by elastic deformation in post above grade.